1993

The College Board

Advanced Placement Examination

CALCULUS AB

SECTION II

Time—1 hour and 30 minutes

Number of problems—6

Percent of total grade—50

SHOW ALL YOUR WORK. Indicate clearly the methods you use because you will be graded on the correctness of your methods as well as on the accuracy of your final answers. If you choose to use decimal approximations, your answer should be correct to three decimal places.

Note: Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers x for which $f(x)$ is a real number.

1. Let f be the function given by $f(x) = x^3 - 5x^2 + 3x + k$, where k is a constant.

   (a) On what intervals is f increasing?

   (b) On what intervals is the graph of f concave downward?

   (c) Find the value of k for which f has 11 as its relative minimum.

2. A particle moves on the x-axis so that its position at any time $t \geq 0$ is given by $x(t) = 2te^{-t}$.

   (a) Find the acceleration of the particle at $t = 0$.

   (b) Find the velocity of the particle when its acceleration is 0.

   (c) Find the total distance traveled by the particle from $t = 0$ to $t = 5$.

3. Consider the curve $y^2 = 4 + x$ and chord AB joining points $A(-4, 0)$ and $B(0, 2)$ on the curve.

   (a) Find the x- and y-coordinates of the point on the curve where the tangent line is parallel to chord AB.

   (b) Find the area of the region R enclosed by the curve and chord AB.

   (c) Find the volume of the solid generated when the region R, defined in part (b), is revolved about the x-axis.

GO ON TO THE NEXT PAGE

4. Let f be the function defined by $f(x) = \ln(2 + \sin x)$ for $\pi \leq x \leq 2\pi$.

   (a) Find the absolute maximum value and the absolute minimum value of f. Show the analysis that leads to your conclusion.

   (b) Find the x-coordinate of each inflection point on the graph of f. Justify your answer.
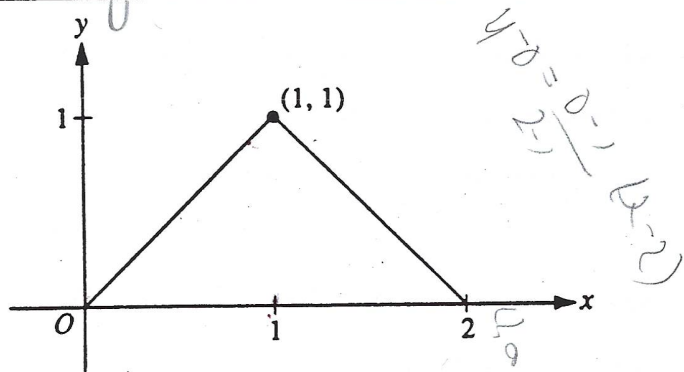
Note: This is the graph of the derivative of f, not the graph of f.

5. The figure above shows the graph of f', the derivative of a function f. The domain of f is the set of all x such that $0 < x < 2$.

   (a) Write an expression for $f'(x)$ in terms of x.

   (b) Given that $f(1) = 0$, write an expression for $f(x)$ in terms of x.

   (c) In the xy-plane provided below, sketch the graph of $y = f(x)$.

   Note: The xy-plane is provided in the pink test booklet only.

6. Let $P(t)$ represent the number of wolves in a population at time t years, when $t \geq 0$. The population $P(t)$ is increasing at a rate directly proportional to $800 - P(t)$, where the constant of proportionality is k.

   (a) If $P(0) = 500$, find $P(t)$ in terms of t and k.

   (b) If $P(2) = 700$, find k.

   (c) Find $\lim\limits_{t \to \infty} P(t)$.

END OF EXAMINATION